Students in PreAP Algebra II will be challenged with an in depth study of Algebra II. **This will require more work outside of class than an on level Algebra II class.** Please be prepared for this work load.

In order for students to be successful in PreAP Algebra II they should be proficient in all objectives from Algebra I. This summer packet has been designed to aid students in **reviewing the content from Algebra I.** The sections in gray are notes and examples of how to do the next section of problems. There are 50 numbered questions to complete.

Students should show all work in the spaces provided in this packet.

Summer packets are due the first day of class.

There will be an assessment over the summer packet material.

Students that are having trouble completing this packet should carefully consider whether they should be in PreAP Algebra II.

Questions or concerns? Email soquinn@hisd.com or rbridges@hisd.com.

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Functions

Identify Functions Relations in which each element of the domain is paired with exactly one element of the range are called **functions**.

Example 1

Determine whether the relation {(6, –3), (4, 1), (7, –2), (–3, 1)} is a function. Explain.

Since each element of the domain is paired with exactly one element of the range, this relation is a function.

Example 2

Determine whether 3*x* – *y* = 6 is a function.

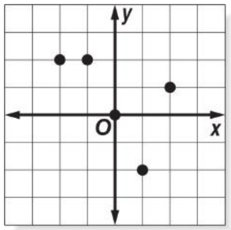
Since the equation is in the form *Ax* + *By* = *C*, the graph of the equation will be a line, as shown at the right.

If you draw a vertical line through each value of *x*, the vertical line passes through just one point of the graph. Thus, the line represents a function.

Exercises

Determine whether each relation is a function.

1.



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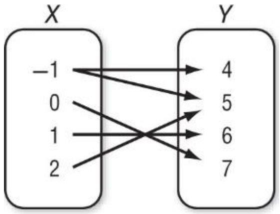

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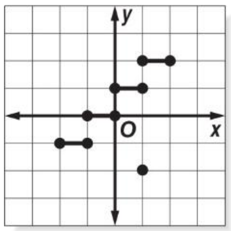

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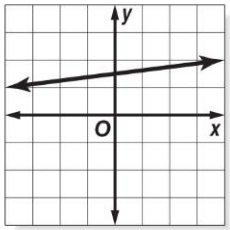


5. {(–3, –3), (–3, 4), (–2, 4)}

Find Function Values Equations that are functions can be written in a form called function notation. For example, $y = 2x - 1$ can be written as $f(x) = 2x - 1$. In the function, *x* represents the elements of the domain, and $f(x)$ represents the elements of the range. Suppose you want to find the value in the range that corresponds to the element 2 in the domain. This is written $f(2)$ and is read "*f* of 2." The value of $f(2)$ is found by substituting 2 for *x* in the equation.

Example: If $f(x) = 3x - 4$, find each value.

a. $f(3)$

$f(3) = 3(3) - 4$	Replace *x* with 3.
$= 9 - 4$	Multiply.
$= 5$	Simplify.

b. $f(-2)$

$f(-2) = 3(-2) - 4$	Replace *x* with –2.
$= -6 - 4$	Multiply.
$= -10$	Simplify.

If $f(x) = 2x - 4$ and $g(x) = x^2 - 4x$, find each value or expression.

6. $f(-5)$

7. $g(k + 1)$

Solve each equation. Check your solution.

8. $5x - 3 = 13 - 3x$

9. $\frac{1}{3}(n + 1) = \frac{1}{6}(3n - 5)$

10. NUMBER THEORY Tripling the greater of two consecutive even integers gives the same result as subtracting 10 from the lesser even integer. What are the integers?

11. GEOMETRY The formula for the perimeter of a rectangle is $P = 2\ell, + 2w$, where ℓ is the length and w is the width. A rectangle has a perimeter of 24 inches. Find its dimensions if its length is 3 inches greater than its width.

Solve for Variables Sometimes you may want to solve an equation such as $V = \ell wh$ for one of its variables. For example, if you know the values of V, w, and h, then the equation $\ell = \frac{V}{wh}$ is more useful for finding the value of ℓ. If an equation that contains more than one variable is to be solved for a specific variable, use the properties of equality to isolate the specified variable on one side of the equation.

Example 1: Solve $2x - 4y = 8$, for y.

$2x - 4y = 8$

$2x - 4y - 2x = 8 - 2x$

$-4y = 8 - 2x$

$\frac{-4y}{-4} = \frac{8 - 2x}{-4}$

$y = \frac{8 - 2x}{-4}$ or $\frac{2x - 8}{4}$

The value of y i s $\frac{2x-8}{4}$.

Example 2: Solve $3m - n = km - 8$, for m.

$3m - n = km - 8$

$3m - n - km = km - 8 - km$

$3m - n - km = -8$

$3m - n - km + n = -8 + n$

$3m - km = -8 + n$

$m(3 - k) = -8 + n$

$\frac{m(3-k)}{3-k} = \frac{-8+n}{3-k}$

$m = \frac{-8+n}{3-k}$ or $\frac{n-8}{3-k}$

The value of m is $\frac{n-8}{3-k}$. Since division by 0 is undefined, $3 - k \neq 0$, or $k \neq 3$.

Solve each equation or formula for the variable indicated.

12. $ax - b = c$, for x

13. $4(r + 3) = t$, for r

14. $16w + 4x = y$, for x

15. $A = \frac{h(a+b)}{2}$, for h

16. $P = 2\ell + 2w$, for w

Point-Slope Form

Point-Slope Form	$y - y_1 = m(x - x_1)$, where (x_1, y_1) is a given point on a nonvertical line and m is the slope of the line

Example 1: Write an equation in point-slope form for the line that passes through (6, 1) with a slope of $-\frac{5}{2}$.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 1 = -\frac{5}{2}(x - 6)$ $m = -\frac{5}{2}$; $(x_1, y_1) = (6, 1)$

Therefore, the equation is $y - 1 = -\frac{5}{2}(x - 6)$.

Example 2: Write an equation in point-slope form for a horizontal line that passes through (4, –1).

$y - y_1 = m(x - x_1)$ Point-slope form

$y - (-1) = 0(x - 4)$ $m = 0$; $(x_1, y_1) = (4, -1)$

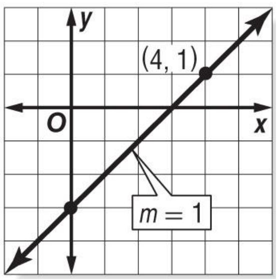
$y + 1 = 0$ Simplify.

Therefore, the equation is $y + 1 = 0$.

Exercises

Write an equation in <u>point-slope form</u> for the line that passes through each point with the given slope.

17.



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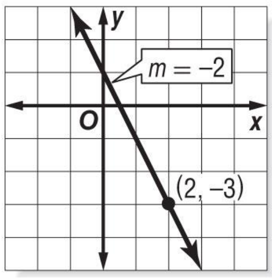


19. $(-4, -5)$, $m = -\frac{1}{2}$

20. Write an equation for a vertical line that passes through (4, –2).

Solve Inequalities To solve linear inequalities involving more than one operation, undo the operations in reverse of the order of operations, just as you would solve an equation with more than one operation.

Example 1: Solve $6x - 4 \leq 2x + 12$.

$6x - 4 \leq 2x + 12$	Original inequality
$6x - 4 - 2x \leq 2x + 12 - 2x$	Subtract 2*x* from each side.
$4x - 4 \leq 12$	Simplify.
$4x - 4 + 4 \leq 12 + 4$	Add 4 to each side.
$4x \leq 16$	Simplify.
$\frac{4x}{4} \leq \frac{16}{4}$	Divide each side by 4.
$x \leq 4$	Simplify.

The solution is $\{x \mid x \leq 4\}$.

Example 2: Solve $3a - 15 > 4 + 5a$.

$3a - 15 > 4 + 5a$	Original inequality
$3a - 15 - 5a > 4 + 5a - 5a$	Subtract *5a* from each side.
$-2a - 15 > 4$	Simplify.
$-2a - 15 + 15 > 4 + 15$	Add 15 to each side.
$-2a > 19$	Simplify
$\frac{-2a}{-2} < \frac{19}{-2}$	Divide each side by –2
$a < -9\frac{1}{2}$	Simplify.

The solution is $\{a \mid a < -9\frac{1}{2}$

Solve each inequality. Check your solution.

21. $\frac{q}{7} + 1 > -5$

22. $\frac{-3x+6}{2} \leq 12$

23. $8 - 2(b + 1) < 12 - 3b$

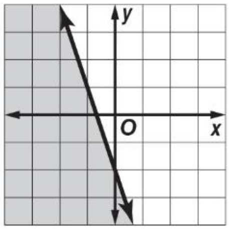
Define a variable, write an inequality, and solve each problem. Check your solution.

24. One fourth of a number decreased by three is at least two

25. Twice the difference of a number and four is less than the sum of the number and five.

Graph Linear Inequalities The solution set of an inequality that involves two variables is graphed by graphing a related linear equation that forms a boundary of a **half-plane**. The graph of the ordered pairs that make up the solution set of the inequality fill a region of the coordinate plane on one side of the half–plane.

Example: Graph $y \le -3x - 2$.



Graph $y = -3x - 2$.

Since $y \le -3x - 2$ is the same as $y < -3x - 2$ and $y = -3x - 2$, the boundary is included in the solution set and the graph should be drawn as a solid line.

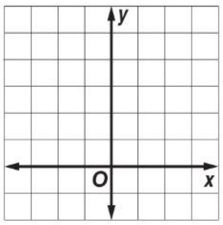
Select a point in each half plane and test it. Choose (0, 0) and (–2, –2).

$y \le -3x - 2$	$y \le -3x - 2$
$0 \le -3(0) - 2$	$-2 \le -3(-2) - 2$
$0 \le -2$ is false.	$-2 \le 6 - 2$
	$-2 \le 4$ is true.

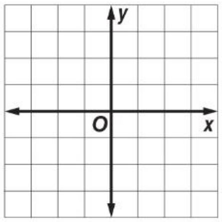
The half-plane that contains (–2, –2) contains the solution. Shade that half-plane.

Graph each inequality.

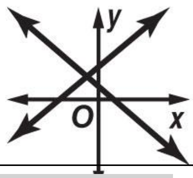
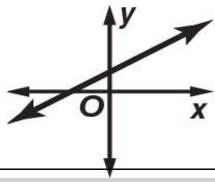
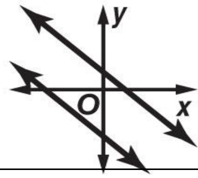
26. $y < 4$



27. $2x - 3y \le 6$



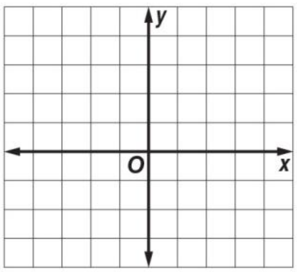
Possible Number of Solutions Two or more linear equations involving the same variables form a **system of equations**. A solution of the system of equations is an ordered pair of numbers that satisfies both equations. The table below summarizes information about systems of linear equations.

Graph of a System	intersecting lines 	same line 	parallel lines 
Number of Solutions	exactly one solution	infinitely many solutions	no solution

Graph each system and determine the number of solutions it has. If it has one solution, name it.

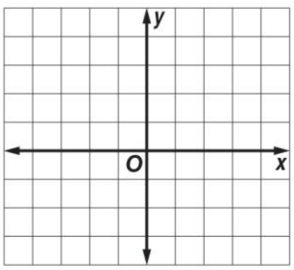
28. $3x + 2y = 6$

$3x + 2y = -4$



29. $2y = -4x + 4$

$y = -2x + 2$



Solve by Substitution One method of solving systems of equations is **substitution.**

Example 1: Use substitution to solve the system of equations.

$y = 2x$
$4x - y = -4$

Substitute $2x$ for y in the second equation.

$4x - y = -4$	Second equation
$4x - 2x = -4$	$y = 2x$
$2x = -4$	Combine like terms.
$x = -2$	Divide each side by 2 and simplify.

Use $y = 2x$ to find the value of y.

$y = 2x$	First equation
$y = 2(-2)$	$x = -2$
$y = -4$	Simplify.

The solution is $(-2, -4)$.

Example 2: Solve for one variable, then substitute.

$x + 3y = 7$
$2x - 4y = -6$

Solve the first equation for x since the coefficient of x is 1.

$x + 3y = 7$	First equation
$x + 3y - 3y = 7 - 3y$	Subtract $3y$ from each side.
$x = 7 - 3y$	Simplify.

Find the value of y by substituting $7 - 3y$ for x in the second equation.

$2x - 4y = -6$	Second equation
$2(7 - 3y) - 4y = -6$	$x = 7 - 3y$
$14 - 6y - 4y = -6$	Distributive Property
$14 - 10y = -6$	Combine like terms.
$14 - 10y - 14 = -6 - 14$	Subtract 14 from each side.
$-10y = -20$	Simplify.
$y = 2$	Divide each side by -10 and simplify.

Use $y = 2$ to find the value of x.

$x = 7 - 3y$

$x = 7 - 3(2)$

$x = 1$

The solution is $(1, 2)$

Exercises: Use substitution to solve each system of equations.

30. $2b = 6a - 14$
$3a - b = 7$

31. $x + y = 16$
$2y = -2x + 2$

Elimination Using Multiplication Some systems of equations cannot be solved simply by adding or subtracting the equations. In such cases, one or both equations must first be multiplied by a number before the system can be solved by elimination.

Example 1: Use elimination to solve the system of equations.

$\boldsymbol{x + 10y = 3}$
$\boldsymbol{4x + 5y = 5}$

If you multiply the second equation by –2, you can eliminate the *y* terms.

$$\begin{aligned} x + 10y &= 3 \\ (+)\ \underline{-8x - 10y} &\underline{= -10} \\ -7x \qquad &= -7 \end{aligned}$$

$$\frac{-7x}{-7} = \frac{-7}{-7}$$

$$x = 1$$

Substitute 1 for *x* in either equation.

$$1 + 10y = 3$$

$$1 + 10y - 1 = 3 - 1$$

$$10y = 2$$

$$\frac{10y}{10} = \frac{2}{10}$$

$$y = \frac{1}{5}$$

The solution is $\left(1, \frac{1}{5}\right)$.

Example 2: Use elimination to solve the system of equations.

$\boldsymbol{3x - 2y = -7}$
$\boldsymbol{2x - 5y = 10}$

If you multiply the first equation by 2 and the second equation by –3, you can eliminate the *x* terms.

$$\begin{aligned} 6x - 4y &= -14 \\ (+)\ \underline{-6x + 15y} &\underline{= -30} \\ 11y &= -44 \end{aligned}$$

$$\frac{11y}{11} = -\frac{44}{11}$$

$$y = -4$$

Substitute –4 for *y* in either equation.

$$\begin{aligned} 3x - 2(-4) &= -7 \\ 3x + 8 &= -7 \end{aligned}$$

$$3x + 8 - 8 = -7 - 8$$

$$3x = -15$$

$$\frac{3x}{3} = -\frac{15}{3}$$

$$x = -5$$

The solution is (–5, –4).

Exercises

Use elimination to solve each system of equations.

32. $2x + 3y = 6$
$x + 2y = 5$

33. GARDENING The length of Sally's garden is 4 meters greater than 3 times the width. The perimeter of her garden is 72 meters. What are the dimensions of Sally's garden

Multiply Monomials A **monomial** is a number, a variable, or the product of a number and one or more variables with nonnegative integer exponents. An expression of the form x^n is called a **power** and represents the product you obtain when *x* is used as a factor *n* times. To multiply two powers that have the same base, add the exponents.

Product of Powers	For any number *a* and all integers *m* and *n*, $a^m \cdot a^n = a^{m+n}$.

Example 1: Simplify $(3x^6)(5x^2)$.

$(3x^6)(5x^2) = (3)(5)(x^6 \cdot x^2)$	Group the coefficients and the variables
$= (3 \cdot 5)(x^{6+2})$	Product of Powers
$= 15x^8$	Simplify.

The product is $15x^8$.

Example 2: Simplify $(–4a^3b)(3a^2b^5)$.

$(–4a^3b)(3a^2b^5) = (–4)(3)(a^3 \cdot a^2)(b \cdot b^5)$

$= –12(a^{3+2})(b^{1+5})$

$= –12a^5b^6$

The product is $–12a^5b^6$.

An expression of the form $(x^m)^n$ is called a **power of a power** and represents the product you obtain when x^m is used as a factor *n* times. To find the power of a power, multiply exponents.

Power of a Power	For any number *a* and any integers *m* and *p*, $(a^m)^p = a^{mp}$.
Power of a Product	For any numbers *a* and *b* and any integer *m*, $(ab)^m = a^m b^m$.

We can combine and use these properties to simplify expressions involving monomials.

Example: Simplify $(-2ab^2)^3(a^2)^4$.

$(-2ab^2)^3\ (a^2)^4 = (-2ab^2)^3(a^8)$	Power of a Power
$= (-2)^3(a^3)\ (b^2)^3\ (a^8)$	Power of a Product
$= (-2)^3(a^3)(a^8)\ (b^2)^3$	Group the coefficients and the variables
$= (-2)^3(a^{11})\ (b^2)^3$	Product of Powers
$= –8a^{11}b^6$	Power of a Power

The product is $–8a^{11}b^6$.

Divide Monomials To divide two powers with the same base, subtract the exponents.

Quotient of Powers	For all integers *m* and *n* and any nonzero number *a*, $\frac{a^m}{a^n} = a^{m-n}$.
Power of a Quotient	For any integer *m* and any real numbers *a* and *b*, *b* ≠ 0, $\left(\frac{a}{b}\right)^m = \frac{a^m}{a^m}$.

Example1: Simplify $\frac{a^4b^7}{ab^2}$. Assume that no denominator equals zero.

$\frac{a^4b^7}{ab^2} = \left(\frac{a^4}{a}\right)\left(\frac{b^7}{b^2}\right)$ Group powers with the same base.

$= (a^{4-1})(b^{7-2})$ Quotient of Powers

$= a^3b^5$ Simplify.

The quotient is a^3b^5

Example 2: Simplify $\left(\frac{2a^3b^5}{3b^2}\right)^3$. Assume that no denominator equals zero.

$\left(\frac{2a^3b^5}{3b^2}\right)^3 = \frac{(2a^3b^5)^3}{(3b^2)^3}$ Power of a Quotient

$= \frac{2^3(a^3)^3(b^5)^3}{(3)^3(b^2)^3}$ Power of a Product

$= \frac{8a^9b^{15}}{27b^6}$ Power of a Power

$= \frac{8a^9b^9}{27}$ Quotient of Powers

The quotient is $\frac{8a^9b^9}{27}$

Negative Exponents Any nonzero number raised to the zero power is 1; for example, $(-0.5)^0 = 1$. Any nonzero number raised to a negative power is equal to the reciprocal of the number raised to the opposite power; for example, $6^{-3} = \frac{1}{6^3}$. These definitions can be used to simplify expressions that have negative exponents..

Zero Exponent	For any nonzero number *a*, $a^0 = 1$.
Negative Exponent Property	For any nonzero number *a* and any integer *n*, $a^{-n} = \frac{1}{a^n}$ *n* and $\frac{1}{a^{-n}} = a^n$.

The simplified form of an expression containing negative exponents must contain only positive exponents.

Example: Simplify $\frac{4a^{-3}b^6}{16a^2b^6c^{-5}}$. Assume that no denominator equals zero.

$\frac{4a^{-3}b^6}{16a^2b^6c^{-5}} = \left(\frac{4}{16}\right)\left(\frac{a^{-3}}{a^2}\right)\left(\frac{b^6}{b^6}\right)\left(\frac{1}{c^{-5}}\right)$ Group powers with the same base.

$= \frac{1}{4}(a^{-3-2})(b^{6-6})(c^5)$ Quotient of Powers and Negative Exponent Properties

$= \frac{1}{4}a^{-5}b^0c^5$ Simplify.

$= \frac{1}{4}\left(\frac{1}{a^5}\right)(1)c^5$ Negative Exponent and Zero Exponent Properties

$= \frac{c^5}{4a^5}$ Simplify.

The solution is $\frac{c^5}{4a^5}$.

Simplify each expression.

34. $(5a^2bc^3)(\frac{1}{5}abc^4)$

35. $-3(ab^4)^3$

36. $(-3j^2k^3)^2(2j^2k)^3$

37. $\frac{xy^6}{y^4x}$

38. $\frac{p^{-8}}{p^3}$

39. $\frac{(6a^{-1}b)^2}{(b^2)^4}$

40. $\left(\frac{4m^2n^2}{8m^{-1}\ell}\right)^0$

Solve Quadratic Equations by Factoring: The following property, along with factoring, can be used to solve quadratic equations.

Zero Product Property	For any real numbers *a* and *b*, if *ab* = 0, then either *a* = 0, *b* = 0, or both *a* and *b* equal 0.

Example : Solve $9x^2 + x = 0$. Then check the solutions.

Write the equation so that it is of the form *ab* = 0.

$9x^2 + x = 0$ Original equation

$x(9x + 1) = 0$ Factor the GCF of $9x^2 + x$, which is x.

$x = 0$ or $9x + 1 = 0$ Zero Product Property

$x = 0$ $x = -\frac{1}{9}$ Solve each equation.

The solution set is $\left\{0, -\frac{1}{9}\right\}$.

Check Substitute 0 and $-\frac{1}{9}$ for x in the original equation.

$9x^2 + x = 0$ $9x^2 + x = 0$

$9(0)^2 + 0 \stackrel{?}{=} 0$ $9\left(-\frac{1}{9}\right)^2 + \left(-\frac{1}{9}\right) \stackrel{?}{=} 0$

$0 = 0$ ✓ $\frac{1}{9} + \left(-\frac{1}{9}\right) \stackrel{?}{=} 0$ $0 = 0$ ✓

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve some equations of the form $ax^2 + bx + c = 0$.

Example: Solve $12x^2 + 3x = 2 - 2x$. Check your solutions.

$12x^2 + 3x = 2 - 2x$	Original equation
$12x^2 + 5x - 2 = 0$	Rewrite equation so that one side equals 0.
$(3x + 2)(4x - 1) = 0$	Factor the left side.
$3x + 2 = 0$ or $4x - 1 = 0$	Zero Product Property
$x = -\frac{2}{3}$ $\quad x = \frac{1}{4}$	Solve each equation.

The solution set is $\left\{-\frac{2}{3}, \frac{1}{4}\right\}$.

Since $12\left(-\frac{2}{3}\right)^2 + 3\left(-\frac{2}{3}\right) = 2 - 2\left(-\frac{2}{3}\right)$ and $12\left(\frac{1}{4}\right)^2 + 3\left(\frac{1}{4}\right) = 2 - 2\left(\frac{1}{4}\right)$, the solutions check.

Differences of Squares

Solve Equations by Factoring Factoring and the Zero Product Property can be used to solve equations that can be written as the product of any number of factors set equal to 0.

Example: Solve each equation. Check your solutions.

a. $x^2 - \frac{1}{25} = 0$

$x^2 - \frac{1}{25} = 0$	Original equation
$x^2 - \left(\frac{1}{5}\right)^2 = 0$	$x^2 = x \cdot x$ and $\frac{1}{25} = \left(\frac{1}{5}\right)\left(\frac{1}{5}\right)$
$\left(x + \frac{1}{5}\right)\left(x - \frac{1}{5}\right) = 0$	Factor the difference of squares.
$x + \frac{1}{5} = 0$ or $x - \frac{1}{5} = 0$	Zero Product Property
$x = -\frac{1}{5}$ $\quad x = \frac{1}{5}$	Solve each equation.

The solution set is $\left\{-\frac{1}{5}, \frac{1}{5}\right\}$. Since $\left(\frac{1}{5}\right)^2 - \frac{1}{25} = 0$ and $\left(\frac{1}{5}\right)^2 - \frac{1}{25} = 0$, the solutions check.

b. $4x^3 = 9x$

$4x^3 = 9x$	Original equation
$4x^3 - 9x = 0$	Subtract $9x$ from each side.
$x(4x^2 - 9) = 0$	Factor out the GCF of x.
$x[(2x)^2 - 3^2] = 0$	$4x^2 = 2x \cdot 2x$ and $9 = 3 \cdot 3$
$x[(2x)^2 - 3^2] = x[(2x - 3)(2x + 3)]$	Factor the difference of squares.
$x = 0$ or $(2x - 3) = 0$ or $(2x + 3) = 0$	Zero Product Property
$x = 0$ $\quad x = \frac{3}{2}$ $\quad x = -\frac{3}{2}$	Solve each equation.

The solution set is $\left\{0, \frac{3}{2}, -\frac{3}{2}\right\}$.

Since $4(0)^3 = 9(0)$, $4\left(\frac{3}{2}\right)^3 = 9\left(\frac{3}{2}\right)$, and $4\left(-\frac{3}{2}\right)^3 = 9\left(-\frac{3}{2}\right)$, the solutions check.

Perfect Squares

Solve Equations with Perfect Squares Factoring and the Zero Product Property can be used to solve equations that involve repeated factors. The repeated factor gives just one solution to the equation. You may also be able to use the **Square Root Property** below to solve certain equations.

Square Root Property	For any number $n > 0$, if $x^2 = n$, then $x = \pm\sqrt{n}$.

Example: Solve each equation. Check your solutions.

a. $x^2 - 6x + 9 = 0$

$x^2 - 6x + 9 = 0$	Original equation
$x^2 - 2(3x) + 3^2 = 0$	Recognize a perfect square trinomial.
$(x - 3)(x - 3) = 0$	Factor the perfect square trinomial.
$x - 3 = 0$	Set repeated factor equal to 0.
$x = 3$	Solve.

The solution set is {3}. Since $3^2 - 6(3) + 9 = 0$, the solution checks.

b. $(a - 5)^2 = 64$

$(a - 5)^2 = 64$	Original equation
$a - 5 = \pm\sqrt{64}$	Square Root Property
$a - 5 = \pm 8$	$64 = 8 \cdot 8$
$a = 5 \pm 8$	Add 5 to each side.
$a = 5 + 8$ or $a = 5 - 8$	Separate into 2 equations.
$a = 13$ $\quad$ $a = -3$	Solve each equation.

The solution set is {–3, 13}. Since $(-3 - 5)^2 = 64$ and $(13 - 5)^2 = 64$, the solutions check.

Solve by Completing the Square Since few quadratic expressions are perfect square trinomials, the method of **completing the square** can be used to solve some quadratic equations. Use the following steps to complete the square for a quadratic expression of the form $ax^2 + bx$.

Step 1	Find $\frac{b}{2}$.
Step 2	Find $\left(\frac{b}{2}\right)^2$.
Step 3	Add $\left(\frac{b}{2}\right)^2$ to $ax^2 + bx$.

Example: Solve $x^2 + 6x + 3 = 10$ by completing the square.

$x^2 + 6x + 3 = 10$	Original equation
$x^2 + 6x + 3 - 3 = 10 - 3$	Subtract 3 from each side.
$x^2 + 6x = 7$	Simplify.
$x^2 + 6x + 9 = 7 + 9$	Since $\left(\frac{6}{2}\right)^2 = 9$, add 9 to each side.
$(x + 3)^2 = 16$	Factor $x^2 + 6x + 9$.
$x + 3 = \pm 4$	Take the square root of each side.
$x = -3 \pm 4$	Simplify.
$x = -3 + 4$ or $x = -3 - 4$	
$= 1$ $\quad$ $= -7$	

The solution set is {–7, 1}.

Quadratic Formula To solve the standard form of the quadratic equation, $ax^2 + bx + c = 0$, use the **Quadratic Formula**.

Quadratic Formula	The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example 1: Solve $x^2 + 2x = 3$ by using the Quadratic Formula.

Rewrite the equation in standard form.

$x^2 + 2x = 3$ Original equation

$x^2 + 2x - 3 = 3 - 3$ Subtract 3 from each side.

$x^2 + 2x - 3 = 0$ Simplify.

Now let $a = 1$, $b = 2$, and $c = -3$ in the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)}$

$= \frac{-2 \pm \sqrt{16}}{2}$

$x = \frac{-2+4}{2}$ or $x = \frac{-2-4}{2}$

$= 1$ $= -3$

The solution set is $\{-3, 1\}$.

Example 2: Solve $x^2 - 6x - 2 = 0$ by using the Quadratic Formula. Leave answers in simplest radical form.

For this equation $a = 1$, $b = -6$, and $c = -2$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-2)}}{2(1)}$

$= \frac{6 + \sqrt{44}}{2}$

$x = \frac{6 + \sqrt{44}}{2}$ or $x = \frac{6 - \sqrt{44}}{2}$

$x = \frac{6 + 2\sqrt{11}}{2}$ or $x = \frac{6 - 2\sqrt{11}}{2}$

$x = 3 + \sqrt{11}$ or $x = 3 - \sqrt{11}$

Exercises

For problems 40- 46 solve each equation by factoring. Check your solutions.

40. $4y^2 = 28y$ **41.** $8x^2 + 2x - 3 = 0$ **42.** $3b^2 - 18b = 10b - 49$

43. The length of a Charlotte, North Carolina, conservatory garden is 20 yards greater than its width. The area is 300 square yards. What are the dimensions?

44. $\frac{1}{4}x^2 = 25$ **45.** $9x^3 = 25x$ **46.** $16n^2 + 16n + 4 = 0$

Solve each equation by <u>completing the square</u>. Leave answers as fractions and simplify radicals.

47. $x^2 + 8x = 20$

48. $x^2 - 14x = -37$

Exercises

Solve each equation by using the Quadratic Formula. Leave answers in simplest radical form.

49. $2x^2 + 9x + 4 = 0$

50. $x^2 - 10x = -23$